

Oscillations in Stochastic Neural Computational Systems

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Abstract

A network of neurons which is used to get solutions to the shortest path problem is augmented with an inhibitory subnetwork which causes the network to exhibit oscillations in the neural activity. These networks are then investigated to determine how oscillations effect the efficiency of neural stochastic combinatorial optimization.

Keywords: stochastic neurons, oscillations, optimization

Description

Oscillations are a prevalent phenomena in biological neural networks, associated with memory processes (Fell & Axmacher, 2011) and sensory perception (Busch, Dubois, & VanRullen, 2009). More specifically oscillations in the 10 Hz range have been observed in the hippocampus of rats and other animals (Green & Arduini, 1954). These rhythms have been shown to be correlated to the speed at which the animal is moving through the environment and have been argued that they are a result of *what* the animal is doing, and not *why* (Vanderwolf, 1969).

In simulated systems oscillations in network activity can be observed by the interaction of inhibitory and excitatory neurons. To investigate this phenomena further we use a network structure based off of a stochastic neural system designed to solve combinatorial optimization problems (Jonke, Habenschuss, & Maass, 2016). Using the shortest path problem as a standard problem to solve we induce a stochastic spiking neural network to solve the problem, discussed below in the Appendix. With the network created we then investigate how oscillations in the neural activity effect the manner in which the network performs stochastic combinatorial optimization.

In the shortest path problem a multitude of potential path lengths, in terms of number of edges, need to be considered, the shortest path may include two edges or ten, and as a result can result in complex network structure to be flexible enough to deal with the scope of the solution space. The Traveling Salesman Problem, while a more complicated computational problem we know we must visit every node in the graph exactly once except for the starting/ending node and leads to a more straight-forward network construction.

This increase in neurons in the induced spiking network results in a increased chance of a cascade effect of neural activity to the point where the network states will no longer induce a feasible graph for the shortest path. To tackle this problem

an inhibitory subnetwork is created with the intention of moderating the activity of neurons within the network in general. Depending upon the choice of connections and parameters to the network in general various behavior can be observed, see Figure 1. The different methods of finding the solution to the problem have both advantages and drawbacks. We note that oscillating networks tend to find induced graphs which contain the solution to the shortest path faster than non-oscillating networks, although during the upswing of the oscillation the induced graph tends to become more complex than the induced graphs in non-oscillating networks.

In a faster paced environment where a solution to the problem needs to be calculated quickly an oscillating network could be optimal, especially if there are multiple copies of the network running in parallel and looking at the graph intersection of the induced graphs to pare down the end resultant graph to find the shortest path. This could be further investigated by establishing a mechanism that would adjust parameters to increase or decrease the frequency of the oscillations in response to the needs of the agent linked with the network.

Further research will investigate the frequency modulation mechanism and how it impacts the performance of the networks in relation to both the shortest path through dynamic networks as well as calculations for the constrained shortest path problem.

Appendix

Background

Suppose a neural network consists of N stochastic spiking neurons, each of which is modeled by a stochastic process, the shape of the postsynaptic potential is modeled by a rectangle given by,

$$x_l(t) = \begin{cases} 1 & \text{if neuron } l \text{ spiked in } (t - \tau, t] \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Then neuron $k \in \{1, \dots, N\}$ fires a spike in a given time span with probability

$$p_k(t) = \begin{cases} \frac{1}{\tau} \exp(u_k(t)) & \text{if } x_k = 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where,

$$u_k(t) = b_k + \sum_l w_{kl} x_l(t) \quad (3)$$

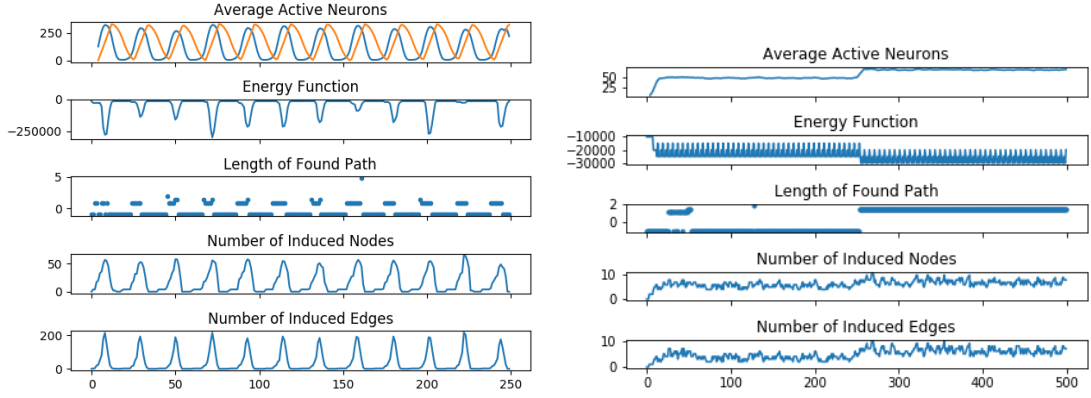


Figure 1: *Left graph*: the construction of the inhibitory subnetwork induced oscillations in the neural activity. *Right graph*: the choice of inhibitory subnetwork has a more gradual descent to the solution. The x -axis represents timesteps in the simulated network, with one timestep representing roughly 1 millisecond.

models the membrane potential of neuron k . In (3), b_k represents the bias of neuron k , which is the base likelihood of firing, w_{kl} is the strength of the connection between neurons k and l , and $x_l(t)$ is the state of neuron l : $x_l(t) = 1$ when the neuron is active and $x_l(t) = 0$ otherwise. The term $w_{kl}x_l(t)$ models the postsynaptic potential (PSP) at time t that resulted from neuron l firing at time t (Jonke et al., 2016).

The energy function of the network is then defined by

$$E_{\mathcal{N}}(x) = -\sum_{k=1}^N b_k x_k - \frac{1}{2} \sum_{k=1}^N \sum_{l=1}^N x_k x_l w_{kl}. \quad (4)$$

It is known that the stochastic spiking neuron model (1)–(2) with the energy function (4) satisfies the *neural computability condition* (Jonke et al., 2016), in which case the stable distribution of the network is given by

$$P(X = x) = \frac{\exp(-E_{\mathcal{N}}(x))}{\sum_{y \in X} \exp(-E_{\mathcal{N}}(y))}. \quad (5)$$

Since lower energy states will have a higher probability of occurrence than higher energy states, the network should be designed in such a way that the optimal solution corresponds to the lowest energy state, thereby increasing the efficiency of the search algorithm. Network structures and principles for constructing networks of stochastic spiking neurons, in particular in application to the TSP, are discussed in (Jonke et al., 2016).

Network Construction

For a given weighted graph $G = (V, E, \phi)$ in the SPP, a corresponding neural network is designed as follows. First, G is augmented with two new vertices: source, S , and target, T , which are connected solely to the source vertex and target vertex in G , respectively, with zero edge weights.

Each $v_i \in V$ is represented by the WTA-cluster C_{v_i} consisting of two identical WTA components, in which each neuron represents a vertex adjacent to v_i . Then with a continuous

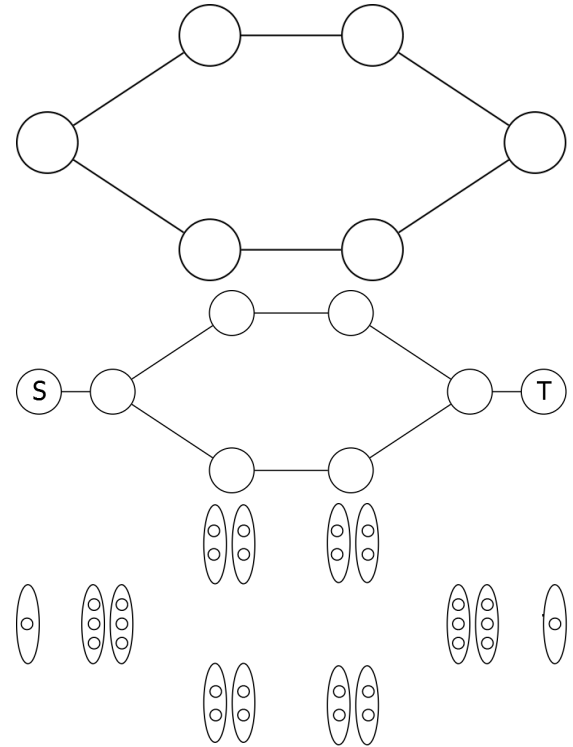


Figure 2: Steps in network construction. *Top Graph*: Starting graph. *Middle Graph*: Addition of source and target nodes. *Bottom Graph*: Construction of general neural architecture. See Figure 3 for final graph construction.

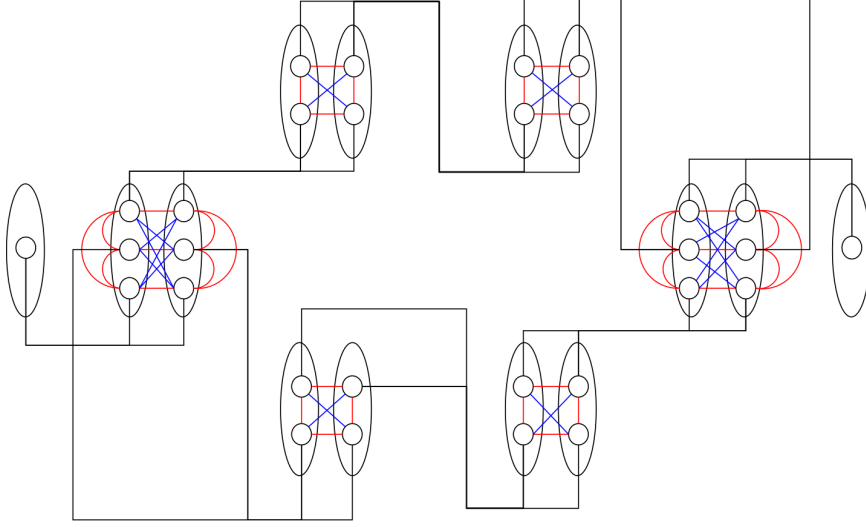


Figure 3: Final connection scheme for the induced stochastic spiking network. Source node on left and target node on right.

function $f : E \times E \rightarrow \mathbb{R}$, the connection strengths of the edges between WTA components is defined by

$$\phi_i(n_j^1, n_k^2) = \begin{cases} f(e(v_i, v_j), e(v_i, v_k)) & \text{if } i \neq j \\ I & \text{if } j = k \end{cases} \quad (6)$$

where $I < 0$ is a constant inhibitory value.

Let

$$f(e_1, e_2) = 1 - \frac{w(e(v_1, v_2)) + w(e(v_2, v_3))}{B}, \quad (7)$$

where B is a constant normalizing value to ensure that $f(e_1, e_2)$ is positive for all edges.

Then each vertex $v_i \in G$ is replaced by C_{v_i} , in which two neurons are equivalent if they represent the same connection in G . In C_{v_i} , neurons are connected in an inhibitory manner to equivalent neurons in the opposing component and connected in an excitatory manner to the neurons in the opposing component for the remaining adjacent vertices. In addition, two single WTA components are added: one for the source S and one for the target T —these are connected solely to their respective WTA component representing the source and target vertices. Let \mathcal{N} denote the constructed neural network.

WTA components C_{v_i} and C_{v_j} representing adjacent vertices v_i and v_j in G are connected as follows. If C_{v_i} has neuron $n_l \in \mathcal{N}$ representing connection to v_j , and C_{v_j} has neuron $n_m \in \mathcal{N}$ representing connection to v_i , then n_l and n_m are connected, and vice versa. The strength of this connection is a fixed parameter $\mathcal{D} > 0$ of the network and is chosen as follows.

Suppose an optimal path contains two vertices v_i and v_j , whose WTA components are C_{v_i} and C_{v_j} , respectively. If neuron $n_j^{i,2}$ becomes active it sends an excitatory signal to both neurons n_i^j , one that becomes active sends an inhibitory signal to the other and an excitatory signal to the other neurons in

the opposite component, this signal should be inversely proportional to the edge weight connecting the respective vertices. Then with the energy function given by Equation 4, the optimal path in G is found as in (Jonke et al., 2016).

The induced graph at time t , $\mathcal{G}(t)$ is the sub-graph that is generated by network state vector $x(t)$. An edge exists between vertices v_i and v_j in the induced graph if in the network the neuron n_i^j is active and neuron n_j^i is active. Note that network states and induced graphs have no one-to-one correspondence, it is possible for different network states to correspond to the same induced graph.

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