A dynamical systems model of intrinsic and evoked activity, variability, and functional connectivity

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Abstract:

Neural signals can be measured experimentally by estimating levels of brain activity, variability, and functional connectivity. However, these neural measures have often been studied independently from one another, making it difficult to infer precise underlying causes of the phenomena. Here we provide a mechanistic framework that relates activity, variability, and functional connectivity in neural mass models. We hypothesized that statistical estimates of activity, variability, and functional connectivity are emergent properties describing network interactions governed by an underlying dynamical system. In testing this hypothesis we provide a dynamical systems mechanism to explain how evoked changes in activity affect changes in moment-to-moment variability and functional connectivity. We demonstrate that a simple network model can reproduce emergent statistical phenomena widely described throughout the task-evoked and dynamic functional connectivity literature. Further, our model suggests that evoked activity shifts the system's attractor dynamics, inducing changes to the moment-to-moment variability and functional connectivity within the network. Together, the proposed mechanisms provide direct links between intrinsic and evoked activity, variability, and functional connectivity under a single dynamical systems framework.

Keywords: functional connectivity; dynamical systems; network modeling; computational modeling

Evoked activity changes moment-to-moment variability and functional connectivity in minimal network models

Many studies have used measures of brain activity, moment-to-moment variability, and functional connectivity (FC) to study brain network properties and brain-behavior relationships. Empirical studies in the functional magnetic resonance imaging (fMRI), electroencephalography (EEG), and magnetoencephalography (MEG) literature have begun to provide evidence that these measures are fundamentally linked and related to behavior (Cocchi, Gollo, Zalesky, & Breakspear, 2017). Similarly, theoretical studies have reported findings that suggest that statistical estimates of intrinsic and evoked activity and variability are highly interrelated (Abbott, Rajan, & Sompolinsky, 2011). Though studies have demonstrated robust relationships between these statistical measures (activity, variability, and FC), a mechanistic understanding of their emergent dynamics is not fully understood.

Here we provide a mechanism that relates changes in observed network statistics that are governed by an underlying dynamical system. Specifically, we relate changes in evoked activity to changes in moment-to-moment variability and FC. To highlight the underlying mechanisms in a simple way and to make our analysis tractable for dynamical systems analysis, we use minimal network models with firing rate dynamics (Wilson & Cowan, 1972). The use of minimal network models allows us to use the tools of dynamical systems theory to perform detailed phase-space analyses, enabling us to visualize the full range of dynamic interactions under various stimulus conditions. The conceptual ideas developed can then be generalized to higher dimensions.

Evoked activation and deactivation reduce moment-to-moment variability in a neural population

Relative to ongoing background activity, evoked activity has widely been associated with reduced moment-to-moment variability. This association has been reported in both large theoretical network models (Abbott et al., 2011) and empirical data across many data modalities and animal models (Cocchi et al., 2017).

One-dimensional neural mass model We used a firing rate model (Wilson & Cowan, 1972) and simulated intrinsic and stimulus-evoked states. We used the simplest model necessary to explain the relationship between evoked activity and moment-to-moment variability, which was a single neural mass population. Specifically, our model was governed by the equation

$$\tau_1 \frac{dx_1}{dt} = -x_1 + f(w_{11}x_1 + b_1 + s_1 + I) \quad (1)$$

where x_i denotes the firing rate (or a measure of activity), τ_i denotes the time constant, w_{ij} refers to the connection strength from node i to j (in this case a self-connection, where $i = j$), b_i refers to the input threshold for optimal activity (or a bias term), s_i refers to the evoked stimulation, I refers to background spontaneous activity sampled from a Gaussian distribution with mean 0 and standard deviation 0.25, and f is the sigmoid input-output activation function

$$f(x) = \frac{1}{1 + e^{-x}} \quad (2)$$

The sigmoid activation function was chosen due to its biological plausibility and previous empirical evidence suggesting that neural masses respond to input in sigmoid-like responses (Rall, 1955). Further, the choice of a sigmoid activation function is intuitive, given that insufficient input into a neural population will fail to excite many (or any) neurons, while extremely strong levels of input can maximally excite all neurons within the population (Wilson & Cowan, 1972).

To demonstrate the relationship between evoked activity and background activity (i.e., spontaneous activity), we simulated the neural population during three states: baseline (spontaneous activity), stimulus evoked de-activation, and stimulus evoked activation. To provide a geometric intuition for these one-dimensional firing rate models, we visualized the phase portrait for each neural population across three simulated states.

Our model accurately demonstrated that evoked activity decreased the moment-to-moment variability of a single neural population. This finding corroborated previous accounts of evoked activity and moment-to-moment variability in both empirical data (He, 2013) and large-scale recurrent network models (Abbott et al., 2011). Further, our minimal model provided us with complete access to the system's dynamics, enabling us to characterize the dynamical mechanisms that underlie changes in the observed output variability. Specifically, we performed a linear stability analysis around the fixed point attractor of the system (i.e., the equilibrium level of activity the system is drawn to during a particular state), and analytically calculated the *characteristic time scale* at the fixed point, which represents the speed with which the system approaches the attractor, and is measured by evaluating the derivative of the system at the fixed point (Fig. 1D) (Strogatz, 1994).

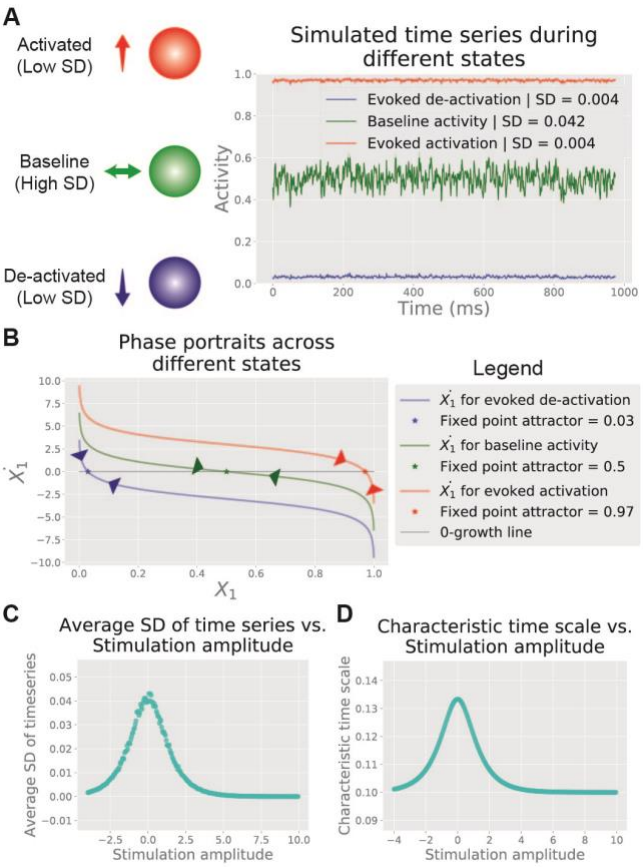

Figure 1: Shifting attractor dynamics provide a mechanism that underlies the relationship between evoked activity and moment-to-moment variability, as estimated with standard deviation (SD). A) For a single neural population, we simulated a baseline state, an activated state, and a de-activated state; for each we measured the SD of the time series. **B)** We visualized the phase portraits for each of the neural populations according to state by plotting the derivative of x_1 (denoted $\dot{x_1}$) by x_1. For each state, we calculated the fixed point attractor (plotted as a star). The arrows denote the direction/vector toward each fixed point. Evoked activity shifts the fixed point attractor, altering the underlying attractor dynamics. **C)** We ran a simulation across a range of stimulation amplitudes, ranging from -4 to 10 in .05 increments. For each stimulation, we calculated the SD across time. **D)** We characterized the shifting attractor dynamics for each state by computing the characteristic time scale at the fixed point for each stimulation amplitude. The characteristic time scale reflects the speed at which the system fluctuates around a given point in the phase space (Strogatz, 1994). The characteristic time scale at a fixed point is perfectly correlated with the measured SD of the time series (rank correlation = 1.0).

Across a range of evoked states, we found that the characteristic time scale explained 100% of the variance of the state-related moment-to-moment variability, providing an explicit dynamical mechanism underlying the relationship between evoked activity and the observed moment-to-moment variability. This is consistent with previous empirical accounts, suggesting that baseline states of spontaneous activity are characterized by slow fluctuations, while evoked states are characterized by fast fluctuations.

Given the biological plausibility of a sigmoid-like function characterizing a neural population, these dynamical mechanisms can be biologically interpreted. Specifically, a baseline state, in which the fixed point attractor is $x_i = 0.5$, the system produces high amplitude, low frequency fluctuations, consistent with previous findings (Cocchi et al., 2017). In contrast, evoked activity brings the fixed point closer to a state of saturation (i.e., close to 1), thereby quenching the moment-to-moment variability due to high input activity causing the neural population to saturate. Lastly, evoked de-activity brings the fixed point closer to a subthreshold state (i.e., close to 0), thereby also quenching the moment-to-moment variability due to low input activity preventing enough elements in the population to fire.

Evoked activity induces changes in functional connectivity in a minimal network model

We next sought to characterize how evoked activity might induce changes in FC. While FC is more generally construed as the statistical dependency between two neural time series, here we operationally define FC be the Pearson correlation between two neural time series. We extend our modeling approach to include two neural populations, the minimal model necessary to interrogate the dynamical mechanisms underlying the relationship between evoked activity and FC.

Two-dimensional neural mass model The model was governed by the equations

$$\tau_1 \frac{dx_1}{dt} = -x_1 + f(w_{11}x_1 + w_{21}x_2 + b_1 + s_1 + I) \quad (3)$$

$$\tau_2 \frac{dx_2}{dt} = -x_2 + f(w_{22}x_2 + w_{12}x_1 + b_2 + s_2 + I) \quad (4)$$

where variables are as described above. All parameter values were identical to the one-dimensional case. The connectivity parameters are taken to be positive.

To quantify the relationship between evoked activity and FC, we systematically simulated the network under different stimulation states. For interpretability, we injected constant, boxcar stimulation to both units, and calculate evoked FC as a function of stimulation amplitude. Notably, given that the injected stimulation is uncorrelated (due to the injected inputs having no variance), it is non-trivial that the FC between two nodes would change across stimulation amplitudes. Relative to the baseline state, evoked activation and de-activation induced decreases in FC between the two neural masses.

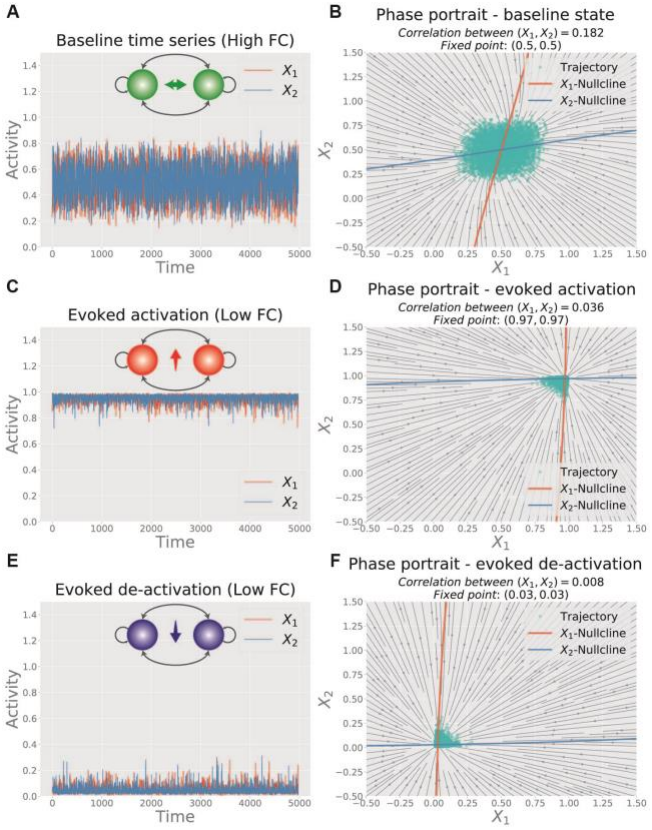

Figure 2: A mechanistic relationship between evoked activity and changes in FC. A) Neural mass time series of the simulated network model in the absence of stimulation. **B)** We plot the phase portraits of our network model to provide a geometric intuition underlying the contribution of dynamics to the emergence of correlations. To plot the vector field, we analytically plotted equation 3 as a function of equation 4. We also plot the trajectory of the system as a scatter plot, to provide an intuition of how correlations might emerge from the underlying vector field. **C)** Neural mass time series during evoked activation. **D)** Phase portrait during a state of evoked activation. Notably, FC decreases from baseline during evoked activation. **E)** Neural mass time series during evoked de-activation. **F)** Phase portrait during a state of evoked de-activation. Notably, FC decreases from baseline during evoked de-activation.

We next performed a phase plane analysis, which allowed us to track the simultaneous evolution of the two variables, thus providing a geometric visualization

of the system (Fig. 2B,D,F). We observed that the baseline state supports trajectories along a diagonal axis (Fig. 2B), which is consistent with the emergence of correlated dynamics between the two units. In contrast, during states with evoked activity, the system approaches the fixed point from all directions in the phase plane more equally, suppressing trajectories along the diagonal axis and thus reducing the overall correlation of the two units (Fig. 2D,F). Thus, changes in FC are associated with changes to the underlying dynamics around the fixed point attractor.

To more rigorously test the relationship between stimulated states and FC, we simulated our network model across a range of stimulation amplitudes. Similar to the one-dimensional model, we found that FC systematically changed (primarily decreases) as a function of the stimulation amplitude. Further, using dynamical systems analysis, we found that a generalization of the characteristic time scale in higher dimensional systems could account for changes in FC as a function of the input stimulation. In other words, we could analytically determine that changes in FC were associated with changes to the underlying vector field as a result of stimulation. Further, we were able to qualitatively replicate these results in a large scale network model, showing that evoked activity induces changes (primarily decreases) in within-network FC. Our findings are qualitatively consistent with empirical results in large scale functional networks with fMRI data (Gonzalez-Castillo & Bandettini, 2017), as well as results observed in large scale network models with chaotic dynamics (Abbott et al., 2011).

Conclusion

Using tools from dynamical systems theory, we provided a mechanistic explanation for the emergence of observed activity, variability, and FC from an underlying dynamical system. We used minimal models to demonstrate that simple network models are sufficient to explain many empirical phenomena widely described throughout the FC literature. Further, the use of minimal models allowed for a tractable dynamical systems analysis to evaluate and probe the system's full dynamics. We then extrapolate the basic principles and mechanisms observed in minimal networks, and show that similar principles are observed in large scale network models (not shown here).

The present theoretical framework establishes a rigorous foundation to interpret statistical measurements from brain networks using mechanisms from dynamical systems theory. Specifically, the framework suggests that observed measures of activity, variability, and FC are highly interrelated and emerge from the same underlying dynamical system. While measures of activity, variability, and FC have often been used independently in previous empirical work to understand brain and cognitive functions, our results suggests that future statistical analyses in empirical data and/or large scale theoretical models should use these measures in combination to triangulate the underlying dynamical states and processes contributing to brain and network function.

Acknowledgments

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